Chapter 3: Solutions to Problems 1-7, 13-14

1. $$q_1 = \frac{q_0(1-sq_0)}{1-sq_0^2}$$

If $q_0 = 0.3,\ s = 0.2,$ and $t = 3$

$$q_1 = \frac{0.3(1-(0.2)(0.3))}{1-(0.2)(0.3)^2} = \frac{0.282}{0.982} = 0.287$$

$$q_2 = \frac{0.287(1-(0.2)(0.287))}{1-(0.2)(0.287)^2} = \frac{0.271}{0.984} = 0.275$$

$$q_3 = \frac{0.275(1-(0.2)(0.275))}{1-(0.2)(0.275)^2} = \frac{0.260}{0.985} = 0.264$$

$$t = \frac{1}{s}\left[\frac{q_0 - q_t}{q_0 q_t} + \ln\frac{q_0(1-q_t)}{q_t(1-q_0)}\right]$$

$$t = \frac{1}{0.2}\left[\frac{0.3-0.264}{(0.3)(0.264)} + \ln\frac{0.3(1-0.264)}{0.264(1-0.3)}\right] = 5(0.455+0.178) = 3.163$$

The approximation in expression 3.6c after 3 generations is roughly 15% greater than that used in 3.6a. The larger the selection coefficient, the greater the difference.

2. $$q_1 = \frac{q_0[1-s(hp_0+q_0)]}{1-2hsp_0q_0 - sq_0^2}$$

If h = 0.0 (selection against recessives)

$$q_1 = \frac{q_0[1-sq_0]}{1-sq_0^2}$$

If h = 0.5 (additivity)

$$q_1 = \frac{q_0[1-s(\frac{1}{2}p_0+q_0)]}{1-2\frac{1}{2}sp_0q_0 - sq_0^2} = \frac{q_0[1-s((\frac{1}{2})(1-q_0)+q_0))]}{1-sp_0q_0-sq_0^2} = \frac{q_0[1-s(\frac{1}{2}-\frac{1}{2}q_0+q_0)]}{1-s(1-q_0)q_0-sq_0^2} =$$

$$q_1 = \frac{q_0[1-s(\frac{1}{2}+\frac{1}{2}q_0)]}{1-(sq_0-sq_0^2)-sq_0^2} = \frac{q_0[1-\frac{1}{2}s(1+q_0)]}{1-sq_0}$$

If h = 1 (selection against dominants)

$$q_1 = \frac{q_0[1-s(p_0+q_0)]}{1-2sp_0q_0-sq_0^2} = \frac{q_0[1-s((1-q_0)+q_0)]}{1-2sq_0(1-q_0)-sq_0^2} = \frac{q_0[1-s(1-q_0+q_0)]}{1-2s(q_0-q_0^2)-sq_0^2} =$$

$$q_1 = \frac{q_0[1-s(1-q_0+q_0)]}{1-2s(q_0-q_0^2)-sq_0^2} = \frac{q_0[1-s]}{1-2sq_0+2sq_0^2-sq_0^2} = \frac{q_0[1-s]}{1-sq_0(2-q_0)}$$

Derivation

$$\Delta q = \frac{pq[q(w_{22}-w_{12})-p(w_{11}-w_{12})]}{p^2 w_{11}+2pqw_{12}+q^2 w_{22}} = \frac{pq[q(1-(1+hs))-p((1+s)-(1+hs))]}{p^2(1+s)+2pq(1+hs)+q^2} =$$

$$\frac{pq[q(1-1-hs)-p(1+s-1-hs))]}{p^2+2pq+q^2+2hspq+sp^2} = \frac{pq[q(-hs)-p(s-hs)]}{1+2hspq+sp^2} =$$

$$\frac{spq(-qh-p+ph)}{1+2hspq+sp^2} = \frac{spq(-(1-p)h-p+ph)}{1+2hspq+sp^2} = \frac{spq(-h+ph-p+ph)}{1+2hspq+sp^2} =$$

$$\frac{spq[-h-p(1-2h)]}{1+2hspq+sp^2} = \frac{-spq[h+p(1-2h)]}{1+2hspq+sp^2}$$

3. $w_{RR} = 0.37$
$w_{RS} = 1.0$
$w_{SS} = 0.68$

Heterozygote Advantage

$s_R = 1 - w_{RR} = 0.63$

$s_S = 1 - w_{SS} = 0.32$

$$q_e = \frac{s_1}{s_1+s_2} = \frac{s_R}{s_R+s_S} = \frac{0.63}{0.63+0.32} = 0.663$$

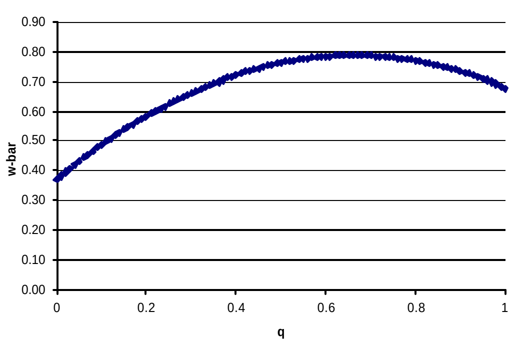
$$\Delta q = \frac{-pq(s_1+s_2)(q-q_e)}{\overline{w}}$$

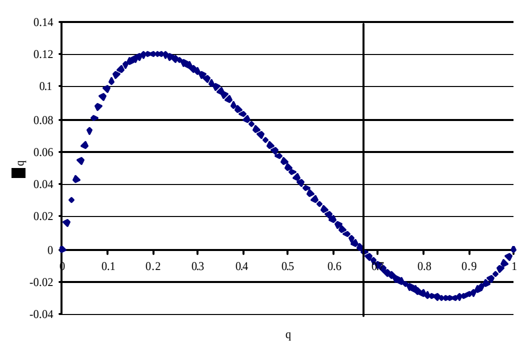
If $q_1 = 0$, $\Delta q = 0$, $q_{eventual} = 0$

If $q_1 = 0.3$, $q_e = 0.663$, $q_{eventual} = 0.663$

If $q_1 = 0.7$, $q_e = 0.663$, $q_{eventual} = 0.663$

If $q_1 = 1$, $\Delta q = 0$, $q_{eventual} = 1$





4. $$q_i = \frac{p_0 q_0 w_{12} + q_0^2 w_{22}}{\overline{w}} \text{ where } \overline{w} = p_0^2 w_{11} + 2 p_0 q_0 w_{12} + q_0^2 w_{22}$$

$$q_i = \frac{pq(1-s) + q^2(1-s)^2}{p^2(1) + 2pq(1-s) + q^2(1-s)^2}$$

$$q_i = \frac{(1-q)q(1-s) + q^2(1-s)^2}{(1-q)^2 + 2(1-q)q(1-s) + q^2(1-s)^2}$$

$$q_i = \frac{(1-s)\left[q(1-q) + q^2(1-s)\right]}{1-2q+q^2+2(q-q^2)(1-s)+q^2(1-2s+s^2)}$$

$$q_i = \frac{(1-s)\left[q - q^2 + q^2 - sq^2\right]}{1-2q+q^2+2q-2sq-2q^2+2sq^2+q^2-2sq^2+s^2q^2}$$

$$q_i = \frac{(1-s)\left[q - sq^2\right]}{1-2sq+s^2q^2}$$

$$q_i = \frac{q(1-s)\left[1-sq\right]}{(1-sq)(1-sq)}$$

$$q_i = \frac{q(1-s)}{1-sq}$$ Equivalent to a haploid model.

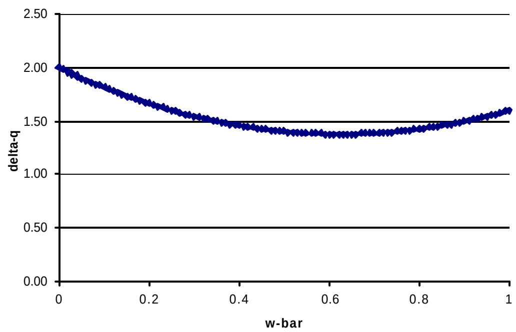
5. $w_{11} = 1$ → 2
$w_{12} = 0.5$ → 1
$w_{22} = 0.8$ → 1.6

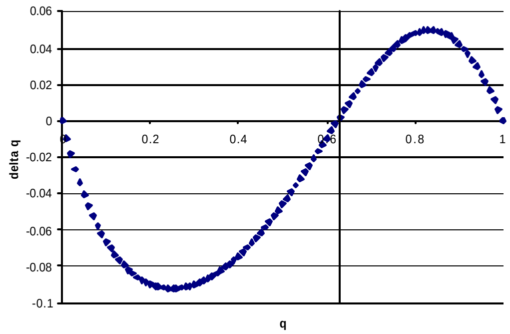
Heterozygote Disadvantage
$s_1 = w_{11} - 1 = 2 - 1 = 1.0$
$s_2 = w_{22} - 1 = 1.6 - 1 = 0.6$

$$q_e = \frac{s_1}{s_1 + s_2} = \frac{1.0}{1.0 + 0.6} = 0.625$$

$$\Delta q = \frac{pq(s_1 + s_2)(q - q_e)}{\overline{w}}$$

If $q_1 = 0$, $\Delta q = 0$, $q_{eventual} = 0$
If $q^1 = 0.3, \Delta q < 0$, $q_{eventual} = 0$
If $q^1 = 0.7$ $\Delta q > 0$, $q_{eventual} = 1$
If $q_1 = 1$, $\Delta q = 0$, $q_{eventual} = 1$





6. $$q_{f_{t+1}} = \frac{\frac{1}{2}(p_f q_m + p_m q_f)w_{12} + q_f q_m w_{22}}{\overline{w}_f} \text{ and } q_{m_{t+1}} = \frac{q_f w_2}{\overline{w}_m}$$

$$\overline{w}_f = p_f p_m w_{11} + (p_f q_m + p_m q_f)w_{12} + q_f q_m w_{22} \text{ and } \overline{w}_m = p_f w_1 + q_f w_2$$

$$w_{f_{11}} = 1.0,\ w_{f_{12}} = 1.0,\ w_{f_{22}} = 0.8,\ w_{m_1} = 1.0,\ w_{m_2} = 0.8$$

$$q_m = 0.2,\ q_f = 0.2$$

$$q_{f_{t+1}} = \frac{\frac{1}{2}((0.8)(0.2) + 1(0.8)(0.2)) + 0.8(0.2)(0.2)}{1(0.8)(0.8) + 1((0.8)(0.2) + (0.8)(0.2)) + 0.8(0.2)(0.2)} = \frac{0.192}{0.992} = 0.194$$

$$q_{m_{t+1}} = \frac{(0.2)(0.8)}{1(0.8) + (0.2)(0.8)} = 0.167$$

$$q_{f_{t+2}} = \frac{\frac{1}{2}((0.806)(0.167) + 1(0.833)(0.194)) + 0.8(0.194)(0.167)}{1(0.806)(0.833) + 1((0.806)(0.167) + (0.833)(0.194)) + 0.8(0.194)(0.167)} = \frac{0.174}{0.994} = 0.175$$

$$q_{m_{t+2}} = \frac{(0.194)(0.8)}{1(0.806) + (0.194)(0.8)} = 0.161$$

$$q_{f_{t+3}} = \frac{\frac{1}{2}((0.825)(0.161) + 1(0.839)(0.175)) + 0.8(0.175)(0.161)}{1(0.825)(0.839) + 1((0.825)(0.161) + (0.839)(0.175)) + 0.8(0.161)(0.175)} = \frac{0.162}{0.994} = 0.163$$

$$q_{m_{t+3}} = \frac{(0.175)(0.8)}{1(0.825) + (0.175)(0.8)} = 0.145$$

7. $x_1 = w_{23}^2 - w_{22}w_{33}$

$x_2 = w_{13}^2 - w_{11}w_{33}$

$x_3 = w_{12}^2 - w_{11}w_{22}$

$y_1 = w_{12}w_{13} - w_{11}w_{23}$

$y_2 = w_{23}w_{12} - w_{22}w_{13}$

$y_3 = w_{23}w_{13} - w_{33}w_{12}$

$z_1 = y_2 + y_3 - x_1$

$z_2 = y_1 + y_3 - x_2$

$z_3 = y_1 + y_2 - x_3$

$$p_{1(e)} = \frac{z_1}{z_1 + z_2 + z_3}$$

$$p_{2(e)} = \frac{z_2}{z_1 + z_2 + z_3}$$

$$p_{3(e)} = \frac{z_3}{z_1 + z_2 + z_3}$$

$x_1 = (0.358)^2 - (0.169)(0.483) = 0.0465$

$x_2 = (0.969)^2 - (0.858)(0.483) = 0.5245$

$x_3 = (1.000)^2 - (0.858)(0.169) = 0.8550$

$y_1 = (1.000)(0.969) - (0.858)(0.358) = 0.6618$

$y_2 = (0.358)(1.000) - (0.169)(0.969) = 0.1942$

$y_3 = (0.358)(0.969) - (0.483)(1.000) = -0.1360$

$z_1 = 01942 + (-0.1360) - 0.0465 = 0.0117$

$z_2 = 0.6618 + (-0.1360) - 0.5245 = 0.0013$

$z_3 = 0.6618 + 0.1942 - 0.8550 = 0.0010$

$$p_{1(e)} = \frac{0.0117}{0.014} \cong 0.836$$

$$p_{2(e)} = \frac{0.0013}{0.014} \cong 0.093$$

$$p_{3(e)} = \frac{0.001}{0.014} \cong 0.071$$

13. $\overline{w} = p_0^2 w_{11} + 2p_0 q_0 + q_0^2 w_{22}$

$$\overline{w} = (0.6)^2(0.9) + 2(0.6)(0.4)1 + (0.4)^2(0.6) = 0.324 + 0.48 + 0.96 = 0.9$$

$$w_e = 1 - \frac{s_1 s_2}{s_1 + s_2} \qquad w_e = 1 - \frac{(0.1)(0.4)}{(0.1) + (0.4)} = 0.92$$

$$q_e = \frac{s_1}{s_1 + s_2} \qquad q_e = \frac{0.1}{0.1 + 0.4} = 0.2$$

Yes, the mean fitness increases until it reaches a maximum value at the stable equilibrium. The maximum value of $\overline{w}$ is 0.92 when $q_e = 0.2$

14. If $s_2 = 1$,

$$t = \frac{1}{q_t} - \frac{1}{q_0}$$

$$t = \frac{1}{0.01} - \frac{1}{0.2} = 95 \text{ generations}$$

If $s_2 = 0.2$,

$$t = \frac{1}{s}\left[\frac{q_0 - q_t}{q_0 q_t} + \ln\frac{q_0(1-q_t)}{q_t(1-q_0)}\right]$$

$$t = \frac{1}{0.2}\left[\frac{0.2 - 0.01}{(0.2)(0.01)} + \ln\frac{0.2(1-0.01)}{0.01(1-0.2)}\right] = 491.04 \text{ or } \sim 491 \text{ generations}$$

$$t = \frac{2}{s}\left[\ln\frac{q_0(1-q_t)}{q_t(1-q_0)}\right]$$

$$t = \frac{2}{0.1}\left[\ln\frac{(0.2)(1-0.01)}{0.01(1-0.2)}\right] = 64.18 \text{ or } \sim 64 \text{ generations}$$