Chapter 5: Solutions to Problems 1-12

1. $f = 0.005$

   $q = 0.005$

   $$\frac{Q_f}{q} = 1 + \frac{fp}{q}$$

   $$\frac{Q_f}{q} = 1 + \frac{(0.005)(0.995)}{0.005} = 1.995$$

   The proportion of inbred individuals is nearly double that of a randomly mating population.

2. $S = 0$

   $H = 2pq = 0.3$

   $T = 1 - S$

   $H_{t+1} = 2pq$ or $H_{t+1} = 2Tpq + \frac{1}{2}SH_t$

   $H_{t+1} = (1)(0.3) + \frac{1}{2}(0)(0.3) = 0.3$

   $H_{t+2} = (1)(0.3) + \frac{1}{2}(0)(0.3) = 0.3$

   $S = 0.5$

   $H = 2pq = 0.3$

   $H_{t+1} = 2Tpq + \frac{1}{2}SH_t$

   $H_{t+1} = (0.5)(0.3) + \frac{1}{2}(0.5)(0.3) = 0.225$

   $H_{t+2} = (0.5)(0.3) + \frac{1}{2}(0.5)(0.225) = 0.206$

   $S = 1$

   $H = 2pq = 0.3$

   $H_{t+1} = \frac{1}{2}H_t$ or $H_{t+1} = 2Tpq + \frac{1}{2}SH_t$

   $H_{t+1} = (0)(0.3) + \frac{1}{2}(1)(0.3) = 0.15$

   $H_{t+2} = (0)(0.3) + \frac{1}{2}(1)(0.15) = 0.075$

   Heterozygosity decreases more rapidly as self-fertilization increases (S→1). When there is no self-fertilization (T=1, S=0), genotypic frequencies remain in Hardy-Weinberg proportions.

3. $S_{t_1} = 0.8$

$S_{t_2} = 1$

$S_{t_3} = 0.8$

$S_{t_4} = 1$

$q = 0.5, 2pq = 0.5$

$H_{t+1} = 2Tpq + \frac{1}{2}SH_t$

$H_{t+1} = (0.2)(0.5) + \frac{1}{2}(0.8)(0.5) = 0.3$

$H_{t+2} = (0)(0.5) + \frac{1}{2}(1)(0.3) = 0.15$

$H_{t+3} = (0.2)(0.5) + \frac{1}{2}(0.8)(0.15) = 0.16$

$H_{t+4} = (0)(0.5) + \frac{1}{2}(1)(0.16) = 0.08$

$S_{t_1} = S_{t_2} = S_{t_3} = S_{t_4} = 0.9$

$q = 0.5, 2pq = 0.5$

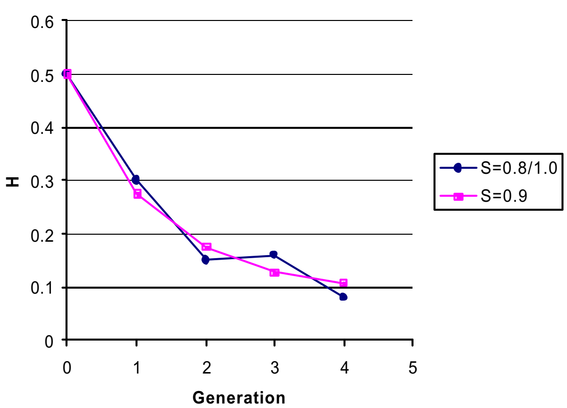
$H_{t+1} = 2Tpq + \frac{1}{2}SH_t$

$H_{t+1} = (0.1)(0.5) + \frac{1}{2}(0.9)(0.5) = 0.275$

$H_{t+2} = (0.1)(0.5) + \frac{1}{2}(0.9)(0.275) = 0.174$

$H_{t+3} = (0.1)(0.5) + \frac{1}{2}(0.9)(0.174) = 0.128$

$H_{t+4} = (0.1)(0.5) + \frac{1}{2}(0.9)(0.128) = 0.108$



4.

$p_i = 0.3$, $p_j = 0.7$, $N_{ij} = 100$, $N = 1000$

$T = \frac{N_{ij}}{P_j N} = \frac{100}{(0.7)(1000)} = 0.143$

$H = Tp_i = 0.143(0.3) = 0.043$

0.043 is the proportion of offspring that are homozygous and outcrossed (See Table 5.4).
The second part of the question asks for the proportion of homozygous progeny that are outcrossed.
Number of outcrossed homozygous progeny = 43
Total number of homozygous progeny = 900
Proportion of homozygous progeny that are outcrossed = 43/900 = 0.048

6. $H_t = 0.5$, selfing

$$H_{t+1} = \frac{1}{2} H_t$$

$$H_{t+1} = \frac{1}{2}(0.5) = 0.25$$

$$H_{t+2} = \frac{1}{2}(0.25) = 0.125$$

$$H_{t+3} = \frac{1}{2}(0.125) = 0.0625$$

$$H_{t+4} = \frac{1}{2}(0.0625) = 0.03125$$

$$H_{t+5} = \frac{1}{2}(0.03125) = 0.0156$$

$H_t = 0.5$, sib-mating

$$H_{t+1} = H_t$$

$$H_{t+2} = \frac{1}{2} H_{t+1} + \frac{1}{4} H_t$$

$$H_{t+1} = 0.5$$

$$H_{t+2} = \frac{1}{2}(0.5) + \frac{1}{4}(0.5) = 0.375$$

$$H_{t+3} = \frac{1}{2}(0.375) + \frac{1}{4}(0.5) = 0.3125$$

$$H_{t+4} = \frac{1}{2}(0.3125) + \frac{1}{4}(0.375) = 0.25$$

$$H_{t+5} = \frac{1}{2}(0.25) + \frac{1}{4}(0.3125) = 0.203$$

$H_t = 0.5$, 1st cousins

$$H_{t+2} = H_{t+1} = H_t$$

$$H_{t+3} = \frac{1}{2} H_{t+2} + \frac{1}{4} H_{t+1} + \frac{1}{8} H_t$$

$$H_{t+1} = 0.5$$

$$H_{t+2} = 0.5$$

$$H_{t+3} = \frac{1}{2}(0.5) + \frac{1}{4}(0.5) + \frac{1}{8}(0.5) = 0.4375$$

$$H_{t+4} = \frac{1}{2}(0.4375) + \frac{1}{4}(0.5) + \frac{1}{8}(0.5) = 0.40625$$

$$H_{t+5} = \frac{1}{2}(0.40625) + \frac{1}{4}(0.4375) + \frac{1}{8}(0.5) = 0.375$$

7. $S_2 = 0.10$

$$f_e = \frac{S_2}{4-3S_2}$$

$$f_e = \frac{0.10}{4-3(0.10)} = 0.027$$

$$\frac{H_{\exp}}{H_{\max}} = \frac{2pq - 2fpq}{2pq} = 1 - f$$

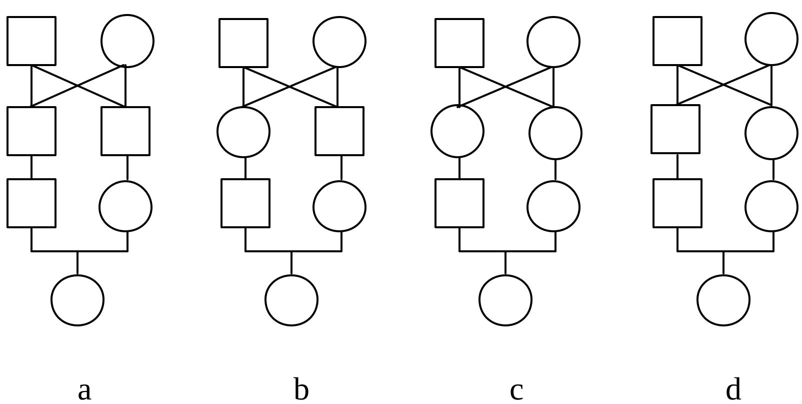
$$\frac{H_{\exp}}{H_{\max}} = 1 - f = 1 - 0.027 = 0.973$$

8.

$$f = \sum_{i=1}^{m} \left(\frac{1}{2}\right)^{N_i} \left(1 + f_{CA_i}\right)$$

$$\left(\tfrac{1}{2}\right)^7 + \left(\tfrac{1}{2}\right)^7 + \left(\tfrac{1}{2}\right)^5 + \left(\tfrac{1}{2}\right)^5 (1+0) = 0.078$$

9.



$$f = \sum_{i=1} \left(\frac{1}{2}\right)^{N_f} \left(1 + f_{CA_i}\right), N_f = \text{\# females in chain}$$

If 2 successive males, the chain is broken.

a) $f = 0$ (2 successive males)

b) $f = \left(\tfrac{1}{2}\right)^3 = 0.125$ (1 chain)

c) $f = \left(\tfrac{1}{2}\right)^3 + \left(\tfrac{1}{2}\right)^4 = 0.1875$ (2 chains)

d) $f = 0$ (2 successive males)

10. $\overline{w} - \overline{w}_f$ = Inbreeding Depression

$$\overline{w} = p^2 w_{11} + 2pqw_{12} + q^2 w_{22}$$

$$\overline{w} = (0.9)^2(1) + 2(0.9)(0.1)(1) + (0.1)(0.8) = 0.998$$

$$\overline{w}_f = w + fqp(w_{11} + w_{22} - 2w_{12})$$

$$\overline{w}_f = 0.998 + (0.25)(0.9)(0.1)(1 + 0.8 - 2(1)) = 0.9935$$

Inbreeding Depression $= 0.998 - 0.9935 = 0.0045$

$\dfrac{\overline{w}_f}{\overline{w}}$ = Relative Fitness for 1 locus

$\dfrac{\overline{w}_f^{500}}{\overline{w}^{500}}$ = Relative Fitness for 500 loci

$$\frac{\overline{w}_f^{500}}{\overline{w}^{500}} = \frac{0.9935^{500}}{0.998^{500}} = 0.104$$

11. $f = 1$

$d = 1 - \dfrac{w_S}{w_O}$ = Inbreeding Depression

$$d = 1 - 0.6 = 0.4$$

$$B = -\frac{1}{f}\ln\left(\frac{w_f}{w_0}\right)$$

$$B = -\frac{1}{1}\ln(0.6) = 0.5108$$

lethal equivalents $= 2B$

lethal equivalents $= 2(0.5108) = 1.02$

12. $w_{11} = 1,\ w_{12} = 0.87,\ w_{22} = 0.7$

$H_0 = 1,\ S = 1$

$$P_1 = \frac{\left[Tp^2 + S(P_0 + \frac{1}{4}H_0)\right]w_{11}}{\overline{w}}$$

$$H_1 = \frac{(2Tpq + \frac{1}{2}SH_0)w_{12}}{\overline{w}}$$

$$Q_1 = \frac{\left[Tq^2 + S(Q_0 + \frac{1}{4}H_0)\right]w_{22}}{\overline{w}}$$

$$\overline{w} = w_{11}\left[Tp^2 + S(P_0 + \tfrac{1}{4}H_0)\right] + w_{12}(2Tpq + \tfrac{1}{2}SH_0) + w_{22}\left[Tq^2 + S(Q_0 + \tfrac{1}{4}H_0)\right]$$

$$\overline{w} = 1\left[0 + 1(0 + \tfrac{1}{4}(1))\right] + 0.87(0 + \tfrac{1}{2}(1)(1)) + 0.7\left[0 + 1(0 + \tfrac{1}{4}(1))\right] = 0.86$$

$$P_{t+1} = \frac{\left[0 + 1(0 + \frac{1}{4}(1))\right]1}{0.86} = 0.291$$

$$H_{t+1} = \frac{(0 + \frac{1}{2}(1)(1))0.87}{0.86} = 0.506$$

$$Q_{t+1} = \frac{\left[0 + 1(0 + \frac{1}{4}(1))\right]0.7}{0.86} = 0.203$$

$A_{2_{t+1}} = q = 0.456$

$$\overline{w} = 1\left[0 + 1(0.291 + \tfrac{1}{4}(0.506))\right] + 0.87(0 + \tfrac{1}{2}(1)(0.506)) + 0.7\left[0 + 1(0.203 + \tfrac{1}{4}(0.506))\right] = 0.868$$

$$P_{t+2} = \frac{\left[0 + 1(0.291 + \frac{1}{4}(0.506)\right]1}{0.868} = 0.481$$

$$H_{t+2} = \frac{(0 + \frac{1}{2}(1)(0.506)0.87}{0.868} = 0.254$$

$$Q_{t+2} = \frac{\left[0 + 1(0.203 + \frac{1}{4}(0.506))\right]0.7}{0.868} = 0.266$$

$A_{2_{t+2}} = q = 0.393$