Chapter 8: Solutions to Problems 1-5, 8, 11

1. $$w = \left(1 - \frac{1}{2}s\right)^{m}$$

$$w = \left(1 - \frac{1}{2}(0.05)\right)^{100} = 0.0795$$

2. Infinite Alleles

$$q = \frac{H_e}{1 - H_e}$$

$$q_X = \frac{0.58}{1 - 0.58} = 1.381\ ,\ q_A = \frac{0.65}{1 - 0.65} = 1.857\ ,\ \frac{q_X}{q_A} = 0.745$$

Stepwise-mutation

$$q = \frac{1}{2}\left[\frac{1}{(1 - H_e)^2} - 1\right]$$

$$q_X = \frac{1}{2}\left[\frac{1}{(1 - 0.58)^2} - 1\right] = 2.334\ ,\ q_A = \frac{1}{2}\left[\frac{1}{(1 - 0.65)^2} - 1\right] = 3.582\ ,\ \frac{q_X}{q_A} = 0.652$$

3. The reason the equilibrium rate of fixation does not involve the population size (N) is that the N cancels out. The overall rate is determined by the product of the probability of fixation of new neutral mutations $(\frac{1}{2N})$ and the average number of new neutral mutations in each generation $(2Nu)$; hence, $(\frac{1}{2N})(2Nu) = u$.
The expected time between neutral substitutions is $\frac{1}{u}$.

$$\frac{1}{u} = \frac{1}{10^{-5}} = 100{,}000 \text{ or } 10^5 \text{ generations}$$

4. $$K_{aa} = -\ln(1 - d_{aa})$$

$$K_{aa} = -\ln\left(1 - \frac{30}{100}\right) = 0.3567$$

$$se(K_{aa}) = \sqrt{\frac{d_{aa}}{(1 - d_{aa})N}},\ \text{where } N = \text{the number of sites examined}$$

$$se(K_{aa}) = \sqrt{\frac{0.3}{(1 - 0.3)100}} = 0.065$$

$$k_{aa} = \frac{K_{aa}}{2T}$$

$$k_{aa} = \frac{0.3567}{2(200{,}000{,}000)} = 8.912x10^{-10}$$

5. $$K = -\frac{3}{4}\ln(1-\frac{4d}{3})$$

$$K = -\frac{3}{4}\ln(1-\frac{4(0.4)}{3}) = 0.572$$

$$se(K) = \sqrt{\frac{d(1-d)}{N(1-\frac{4d}{3})^2}}$$

$$se(K) = \sqrt{\frac{0.4(1-0.4)}{500(1-\frac{4(0.4)}{3})^2}} = 0.0469$$

$$k = \frac{K}{2T}$$

$$k_{aa} = \frac{0.572}{2(5{,}000{,}000)} = 5.72x10^{-8}$$

8. $$\Pr(2)^t[1-\Pr(2)] = \left(1-\frac{1}{2N}\right)^t \frac{1}{2N}$$

$$\Pr(2)^t[1-\Pr(2)] = \left(1-\frac{1}{2(50)}\right)^9 \frac{1}{2(50)} = 0.009$$

$$E(T_n) = \frac{4N}{n(n-1)}$$

$$E(T_n) = \frac{4(5)}{10(10-1)} = 0.222$$

$$E(T_n) = \frac{4(5)}{5(5-1)} = 1$$

$$E(T_n) = \frac{4(5)}{2(2-1)} = 10$$

11. $$\frac{9}{59} = 0.15, \frac{24}{44} = 0.54$$

3.2	5.8	9
20.8	38.2	59
24	44	

$$\frac{12}{41} = 0.29, \frac{35}{18} = 1.94$$

7.9	4.1	12
27.1	13.9	41
35	44	