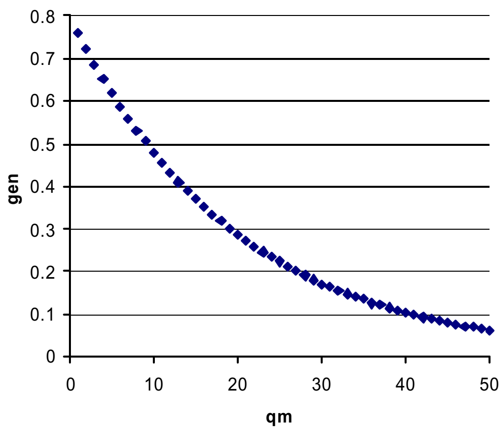
Chapter 9: Solutions to Problems 1-8, Pg 522

1. $q_1 = (1-m)q_0 + mq_m$



2.

$$X = \begin{vmatrix} 0.9850 & 0.0125 & 0.0025 \\ 0.0138 & 0.9775 & 0.0087 \\ 0.0000 & 0.0098 & 0.9902 \end{vmatrix} \qquad Y = \begin{vmatrix} 0.575 \\ 0.567 \\ 0.505 \end{vmatrix}$$

$$Y'_{gen1} = \begin{vmatrix} 0.9850(0.575) + 0.0125(0.567) + 0.0025(0.505) \\ 0.0138(0.575) + 0.9775(0.567) + 0.0087(0.505) \\ 0.0000(0.575) + 0.0098(0.567) + 0.9902(0.505) \end{vmatrix} = \begin{vmatrix} 0.5747_{Nuer} \\ 0.5666_{Dinka} \\ 0.5056_{Shilluk} \end{vmatrix}$$

$$Y'_{gen2} = \begin{vmatrix} 0.9850(0.5747) + 0.0125(0.5666) + 0.0025(0.5056) \\ 0.0138(0.5747) + 0.9775(0.5666) + 0.0087(0.5056) \\ 0.0000(0.5747) + 0.0098(0.5666) + 0.9902(0.5056) \end{vmatrix} = \begin{vmatrix} 0.5744_{Nuer} \\ 0.5661_{Dinka} \\ 0.5052_{Shilluk} \end{vmatrix}$$

$$Y'_{gen3} = \begin{vmatrix} 0.9850(0.5744) + 0.0125(0.5661) + 0.0025(0.5052) \\ 0.0138(0.5744) + 0.9775(0.5661) + 0.0087(0.5052) \\ 0.0000(0.5744) + 0.0098(0.5661) + 0.9902(0.5052) \end{vmatrix} = \begin{vmatrix} 0.5741_{Nuer} \\ 0.5657_{Dinka} \\ 0.5058_{Shilluk} \end{vmatrix}$$

3. Lumped

$$\overline{H} = \frac{2(0.2)(0.8) + 2(0.4)(0.6)}{2} = 0.4, \quad \overline{P} = \frac{0.2^2 + 0.4^2}{2} = 0.1, \quad \overline{Q} = \frac{0.8^2 + 0.6^2}{2} = 0.5$$

Random

$$2pq = 2(0.3)(0.7) = 0.42, \quad p^2 = 0.3^2 = 0.09, \quad q^2 = 0.7^7 = 0.49$$

$$p = \frac{0.2 + 0.4}{2} = 0.3, \quad q = \frac{0.4 + 0.10}{2} = 0.7$$

Lumped

$$A_1A_2 + A_1A_3 + A_2A_3 = 0.2 + 0.2 + 0.2 = 0.6$$

$$A_1A_2 = \frac{2(0.2)(0.4) + 2(0.6)(0.2)}{2} = 0.2,\ A_1A_3 = \frac{2(0.2)(0.4) + 2(0.6)(0.2)}{2} = 0.2,$$

$$A_2A_3 = \frac{2(0.4)(0.4) + 2(0.2)(0.2)}{2} = 0.2$$

$$A_1A_1 = \frac{0.2^2 + 0.6^2}{2} = 0.2,\ A_2A_2 = \frac{0.4^2 + 0.2^2}{2} = 0.1,\ A_3A_3 = \frac{0.4^2 + 0.2^2}{2} = 0.1$$

Random

$$A_1A_2 + A_1A_3 + A_2A_3 = 0.24 + 0.24 + 0.18 = 0.66$$

$$A_1A_2 = 2(0.4)(0.3) = 0.24,\ A_1A_3 = 2(0.4)(0.3) = 0.24,\ A_2A_3 = 2(0.3)(0.3) = 0.18$$

$$A_1 = \frac{0.2 + 0.6}{2} = 0.4,\ A_2 = \frac{0.4 + 0.2}{2} = 0.3,\ A_3 = \frac{0.4 + 0.2}{2} = 0.3$$

$$A_1A_1 = 0.4^2 = 0.16,\ A_2A_2 = 0.3^2 = 0.09,\ A_3A_3 = 0.3^2 = 0.09$$

When subdivided populations are lumped together, there is a deficiency of heterozygotes and an excess of homozygotes relative to a randomly mating population. This is known as the Wahlund effect.

4. $$\hat{M} = \frac{q_A - q_H}{q_A - q_B} = \frac{0.2 - 0.3}{0.2 - 0.6} = 0.25$$

$$\hat{m} = 1 - e^{\ln(x)/t} = 1 - e^{\ln(0.75)/10} = 0.0284$$

'The estimate of gene flow assume that the effects of selection and genetic drift on allelic frequencies are negligible relative to the amount of gene flow.'

5. $$\overline{F}_{IS} = \frac{\overline{H}_S - \overline{H}_0}{\overline{H}_S},\ \overline{F}_{IT} = \frac{\overline{H}_T - \overline{H}_0}{\overline{H}_T},\ \overline{F}_{ST} = \frac{\overline{H}_T - \overline{H}_S}{\overline{H}_T}$$

$$\overline{H}_0 = \frac{0.5 + 0.3}{2} = 0.4,\ \overline{H}_S = \frac{2(0.5)(0.5) + 2(0.5)(0.5)}{2} = 0.5,\ \overline{H}_T = 2(0.5)(0.5) = 0.5$$

$$\overline{F}_{IS} = \frac{0.5 - 0.4}{0.5} = 0.2,\ \overline{F}_{IT} = \frac{0.5 - 0.4}{0.5} = 0.2,\ \overline{F}_{ST} = \frac{0.5 - 0.5}{0.5} = 0.0$$

$$\overline{H}_0 = \frac{0.5 + 0.42}{2} = 0.46,\ \overline{H}_S = \frac{2(0.5)(0.5) + 2(0.3)(0.7)}{2} = 0.46,\ \overline{H}_T = 2(0.4)(0.6) = 0.48$$

$$\overline{F}_{IS} = \frac{0.46 - 0.46}{0.5} = 0.0,\ \overline{F}_{IT} = \frac{0.48 - 0.46}{0.48} = 0.04167,\ \overline{F}_{ST} = \frac{0.48 - 0.46}{0.48} = 0.04167$$

6. $H_S = 1 - 5(0.2^2) = 0.8$

$$G_{ST} = \frac{(k-1)(1-H_S)}{k-1+H_S}$$

$$G_{ST} = \frac{(2-1)(1-0.8)}{(2-1+0.8)} = \frac{0.2}{1.8} = 0.1111$$

This value indicates a low level of genetic differentiation but does not account for the non-overlapping set of alleles.

7. $$F_{SR} = \frac{H_R - H_S}{H_R}$$

$$F_{RT} = \frac{H_T - H_R}{H_T}$$

$$H_R = \frac{2(0.109) + 2(0.469) + 3(0.435)}{7} = 0.3516$$

$$H_S = \frac{0 + 0.204 + 0.401 + 0.024 + 0.411 + 0.366 + 0.434}{7} = 0.2629$$

$$F_{SR} = \frac{0.3516 - 0.2629}{0.3516} = 0.2524$$

$$F_{RT} = \frac{0.647 - 0.3516}{0.647} = 0.4566$$

8. $$F_{ST} = \frac{F_{ST(m)}F_{ST(p)}}{F_{ST(m)} + F_{ST(p)} - 3F_{ST(m)}F_{ST(p)}}$$

$$F_{ST} = \frac{(0.05)(0.5)}{0.05 + 0.5 - 3(0.05)(0.5)} = \frac{0.025}{0.475} = 0.0526$$

The high level of maternal gene flow results in a low overall F_{ST}.

$$\frac{m_p}{m_s} = \frac{F_{ST(m)} - 2F_{ST} + F_{ST}F_{ST(m)}}{F_{ST}(1 - F_{ST(m)})}$$

$$\frac{m_p}{m_s} = \frac{0.5 - 2(0.05) + (0.05)(0.5)}{(0.05)(1-0.5)} = \frac{0.425}{0.025} = 17$$